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Rellich inequalities for polyharmonic operators in plane domains

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Abstract

© 2018 Russian Academy of Sciences (DoM), London Mathematical Society, Turpion Ltd. Functionals whose values are defined as sharp constants in Rellich inequalities are investigated for polyharmonic operators in plane domains. The weight function is taken to be a power of the distance of a point to the boundary of the domain. Estimates are obtained for arbitrary domains, as is a test for these constants to be positive, and precise values are found for convex domains and for domains close to convex in a certain sense. The case when the weight function is taken to be a power of the coefficient in the Poincaré metric is also treated. Bibliography: 28 titles.

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Keywords

Poincare metric, polyharmonic operator, Rellich inequality, uniformly perfect set

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